2143-110 Discrete Mathematic

Topic Quiz 1 4/Mar/2010 16:15 – 17:45 (90 minutes)

Name ____ Sect _____ ID ___________________

Notes

- This paper has 2 question pages and 3 blank answer sheets. **Write down your name and ID on top of every page**.
- Write down your answer **only** in the blank sheet.
- No calculator, closed book, and don't bring any paper in.
- Cheating will not be tolerated.

Here are some useful logical equivalences and set identities.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$
$p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$
$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \leftrightarrow q \equiv (p \to q) \wedge (p \to p)$

$A \cup (B \cap C) \equiv (A \cup B) \cap (A \cup C)$
$A \cap (B \cup C) \equiv (A \cap B) \cup (A \cap C)$
$\overline{A \cap B} \equiv \bar{A} \cup \bar{B}$
$\overline{A \cup B} \equiv \bar{A} \cap \bar{B}$
$A \cup (A \cap B) \equiv A$
$A \cap (A \cup B) \equiv A$
$A \cup \bar{A} = U$
$A \cap \bar{A} = \emptyset$

Here are some useful rules of inference.

p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism

$\forall x P(x)$ $\therefore P(c)$	Universal instantiation
$\exists x P(x)$ $\therefore P(c)$ for some element c	Existential instantiation
$P(c)$ for an arbitrary c $\therefore \forall x P(x)$	Universal generalization
$P(c)$ for some element c $\therefore \exists x P(x)$	Existential generalization

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1. (2 Pts) How many rows appear in a truth table for each of these compound propositions?

   (a) $p \to \neg p$

   (b) $(p \vee \neg r) \wedge (q \vee \neg s)$

   (c) $q \vee p \vee \neg s \vee \neg r \vee \neg t \vee u$

   (d) $(p \wedge r \wedge t) \leftrightarrow (q \wedge t)$

2. (2 Pts) Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x P(x) \vee \forall x Q(x)$ is true.

1.	$\forall x(P(x) \vee Q(x))$	Premise
2.	$P(c) \vee Q(c)$	Universal instantiation from (1)
3.	$P(c)$	Simplification from(2)
4.	$\forall x P(x)$	Universal instantiation from (3)
5.	$Q(c)$	Simplification from(2)
6.	$\forall x Q(x)$	Universal instantiation from (5)
7.	$\forall x P(x) \vee \forall x Q(x)$	Conjunction from (4) and (6)

3. (2 Pts) Given an example of a predicate $P(x, y)$ such that $\exists x \forall y P(x, y)$ and $\forall y \exists x P(x, y)$ have different truth values.

4. (2 Pts) Let $P(m, b)$ be the statement "m divides n," where the domain for both variables consists of all positive integers. (By "m divides n" we mean that $n = km$ for some integer k.) Determine the truth values of each of these statements.

   (a) $P(4, 5)$

   (b) $\exists n \forall m P(m, n)$

   (c) $\forall n P(1, n)$

5. (2 Pts) Find the power set of $\{\emptyset, \{\emptyset\}\}$.

6. (2 Pts) Suppose that A, B, C and D are sets. Prove or disprove that $(A-B)-(C-D) = (A-C)-(B-D)$.

7. (3 Pts) For which real numbers x and y is it true that $\lfloor x+y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$. Be noted that it is not correct to show just only some possible values of x and y, i.e., if you answer "when both x and y are 1", you will get no point. You need to answer a condition (or conditions) that governs all possible values.

8. (3 Pts) Express the statement "There is exactly one student in this class who has taken exactly one mathematic class at this school". The predicate must be $T(m, n)$ which means "m has taken class n". You cannot use uniqueness quantifier.

9. (4 Pts) Proof that $\sqrt{2}$ is irrational by giving a proof by contradiction.

10. (4 Pts) Let $P(n)$ be the statement that $1^2 + 2^2 + \ldots + n^2 = n(n+1)(2n+1)/6$ for the positive integer n. Use mathematical induction to prove $P(n)$ by following these steps.

    (a) What is the statement $P(1)$

    (b) Show that $P(1)$ is true (this is the basis step).

    (c) What is the inductive hypothesis?

    (d) What do you need to prove in the inductive step?

    (e) Complete the inductive step.

    (f) Explain why these steps show that $P(n)$ is true whenever n is a positive integer.

11. (4 Pts) Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$. Hint: The basis steps should be $P(18), P(19), P(20)$ and $P(21)$. After that, use strong induction.

Name ______________________________ Sect _____ ID ________________