

2413-110 Discrete Mathematic

Quiz 1 9/Feb/2011 16:30 – 17:30 (60 minutes)

Name _____ Sect _____ ID _____

Notes

- This paper has 2 question pages and 3 blank sheets. **Write down your name and ID on top of every page.**
 - Write down your answer **only** in the blank sheet.
 - No calculator, closed book, and don't bring any paper in.
 - Cheating will not be tolerated.
 - Here are some useful logical equivalences
 1. $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
 2. $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
 3. $\neg(p \wedge q) \equiv \neg p \vee \neg q$
 4. $\neg(p \vee q) \equiv \neg p \wedge \neg q$
 5. $p \vee (p \wedge q) \equiv p$
 6. $p \wedge (p \vee q) \equiv p$
 - \mathbb{R} is the set of real numbers. \mathbb{N} is the set of integers
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1. State the converse, contrapositive, and inverse of each of these statements.
 - (a) If it snows today, I will ski tomorrow.
 - (b) I come to class whenever there is going to be a quiz.
2. Construct a truth table for each of these compound propositions.
 - (a) $p \rightarrow \neg p$
 - (b) $p \oplus (p \vee q)$
 - (c) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
 - (d) $(p \wedge q) \rightarrow (p \vee q)$
3. Show that $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.
4. The dual of a compound proposition that contains only the logical operators \vee, \wedge , and \neg is the compound proposition obtained by replacing each \vee by \wedge , each \wedge by \vee , each T by F , and each F by T . Find the dual of each of these compound propositions.
 - (a) $p \vee \neg q$
 - (b) $p \wedge (q \vee (r \wedge T))$
 - (c) $(p \wedge \neg q) \vee (q \wedge F)$
5. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y , where x is 1, 2 or 3 and y is 1, 2 or 3. Write out these propositions using \vee and \wedge .

- (a) $\exists xP(x, 3)$
- (b) $\forall yP(1, y)$
- (c) $\exists y\neg P(2, y)$
- (d) $\forall x \neq P(x, 2)$

6. Use set builder notation to give a description of each of these sets.

- (a) $\{0, 3, 6, 9, 12\}$
- (b) $\{-3, -2, 1, 0, 1, 2, 3\}$

7. Determine whether each of these statements are true or false.

- (a) $\emptyset \in \{\emptyset\}$
- (b) $\{\emptyset\} \in \{\emptyset\}$
- (c) $\{\emptyset\} \subset \{\emptyset, \{\emptyset\}\}$
- (d) $\{\{\emptyset\}\} \subset \{\{\emptyset\}, \{\emptyset\}\}$

8. The successor of the set A is the set $A \cup \{A\}$. Find the successors of the following sets.

- (a) $\{1, 2, 3\}$
- (b) \emptyset
- (c) $\{\emptyset\}$
- (d) $\{\emptyset, \{\emptyset\}\}$

9. Give an example of a function whose domain is \mathbb{N} and co-domain is \mathbb{N} that is

- (a) one-to-one but not onto.
- (b) onto but not one-to-one.
- (c) both onto and one-to-one.
- (d) neither one-to-one nor onto.

10. If f and $f \circ g$ are one-to-one, does it follow that g is one-to-one? Justify your answer.