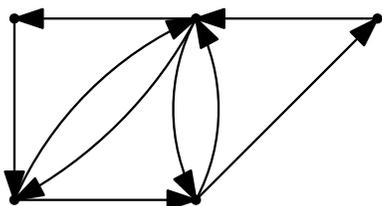
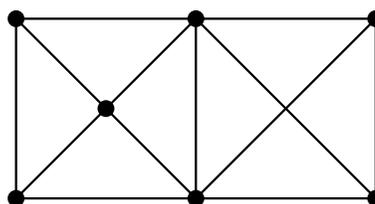


## Notes

- This paper has 1 question page (this page) and 3 blank sheets. **Write down your name and ID on top of every page.**
- Write down your answer **only** in the blank sheet.
- No calculator, closed book, and don't bring any paper in.
- Cheating will not be tolerated.



(a) The graph for problem 3



(b) The graph for problem 6

- (1 point) The **intersection graph** of a collection of sets  $A_1, A_2, \dots, A_n$  is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of this collection of sets  $A_1 = \{2, 4, 6, 8\}$ ,  $A_2 = \{0, 1, 2, 3, 4\}$ ,  $A_3 = \{1, 3, 5, 7, 9\}$ ,  $A_4 = \{5, 6, 7, 8, 9\}$ ,  $A_5 = \{0, 1, 8, 9\}$
- (1 point) Determine whether the graphs without loops with these **incidence matrices** are isomorphic.
 
$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
- (1 points) Determine whether the directed graph in Figure (a) has an Euler circuit. Construct an Euler circuit if one exists.
- (2 points) Show that the chromatic number of any tree does not exceed two.
- (2 points) Suppose that  $T$  is a full  $m$ -ary tree. Let  $i$  be the number of internal vertices,  $l$  be the number of leaves and  $n$  be the number of all vertices. Show that, if  $i$  and  $m$  are given, then  $n = mi + 1$  and  $l = (m - 1)i + 1$ .
- (2 points) A clique in a simple undirected graph is a complete subgraph that is not contained in any larger complete subgraph. Find all cliques in the graph shown in Figure (b).
- (3 points) Construct the ordered rooted tree whose preorder traversal is  $a, b, f, c, g, h, i, d, e, j, k, l$ , where  $a$  has 4 children,  $c$  has 3 children,  $j$  has 2 children,  $b$  and  $e$  have 1 child each, and all other vertices are leaves.
- (3 points) Suppose that  $G_1$  and  $H_1$  are isomorphic and that  $G_2$  and  $H_2$  are isomorphic. **Prove or disprove** that  $G_1 \cup G_2$  and  $H_1 \cup H_2$  are isomorphic.