2413-110 Discrete Mathematic

Quiz 1 30/Jan/2009 11:05 – 12:30 (85 minutes)

Name ______________________________ Sect _____ ID ______________

Notes

- This paper has 1 question page (this page) and 3 blank sheets. **Write down your name and ID on top of every page**.
- Write down your answer **only** in the blank sheet.
- No calculator, closed book, and don't bring any paper in.
- Cheating will not be tolerated.
- Here are some useful logical equivalences
  1. $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$
  2. $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
  3. $\neg(p \land q) \equiv \neg p \lor \neg q$
  4. $\neg(p \lor q) \equiv \neg p \land \neg q$
  5. $p \lor (p \land q) \equiv p$
  6. $p \land (p \lor q) \equiv p$

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1. (1 point) Determine whether $(\neg p \land (p \to q)) \to \neg q$ is a tautology.

2. (1 point) List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$ where $(a, b) \in R$ if and only if

   (a) $a = b$. (b) $a + b = 4$. (c) $a > b$.

3. (2 points) Show that $(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$.

4. (2 points) Express the negation of each of these statements so that all negation symbols immediately precede predicates.

   (a) $\forall x \exists y (P(x, y) \land \exists z R(x, y, z))$ (b) $\forall x \exists y (P(x, y) \to Q(x, y))$

5. (2 points) Show that if n is an integer, then $n = \lceil n/2 \rceil + \lfloor n/2 \rfloor$.

6. (3 points) Suppose that

   $$\mathbf{A} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

   where a and b are real numbers. Show that

   $$\mathbf{A}^n = \begin{bmatrix} a^n & 0 \\ 0 & b^n \end{bmatrix}$$

   for every positive integer n (hint: use mathematical induction.)

7. (4 points) Prove that if n is an integer and $3n + 2$ is even, then n is even using

   (a) an indirect proof. (b) a proof by contradiction.