FACULTY OF ENGINEERING
CHULALONGKORN UNIVERSITY
2110201 Computer Engineering Mathematics
YEAR II, Second Semester, Midterm Examination, January 7, 2014, 16:15 – 18:15

Name________________________________ID | | | | | | | | |2|1| CR58_________

Instructions

1. There are 44 questions, with the total number of 8 pages
2. Write down your name and ID on top of every page.
3. Write down your answer only in the provided space.
4. No documents or calculator are allowed inside the exam room. You must not bring any part of this exam paper outside. This exam paper is a government's property. Violators will be prosecuted under a criminal court.
5. A student who wishes to leave the exam room before the end of the exam period must raise their hand and ask for permission before leaving the room. Student must leave the room in orderly manner.
6. Once the time is expired, students must stop writing and must remain seated quietly until the proctors collect all the exam papers or given exam booklets. Only then, the student will be allowed to leave the room in orderly manner.
7. Any student with an act of cheating according to the Faculty of Engineering Announcement regarding the exam regulations is subjected to receive an F and at least 1 semester academic suspension.

I acknowledge all instructions above.

Student's Signature…………………………………….

Part I

True or False. (One point each. A wrong answer is subject to one point deduction.)

1. ____ It is true that a system of linear equations might not have a solution but its homogeneous system might have infinitely many solutions?
2. ____ We can always find a solution of any homogeneous system.
3. ____ Any matrix in reduced echelon form is also in echelon form
4. ____ A subspace of $M_{2\times 3}$cannot has a basis with more than 6 elements.
5. ____ Intersection of n different subspaces of any vector space V must be a subspace of V.
6. ____ A subset of any trivial space is always a subspace.
7. ____ A non-empty vector space must contain infinitely many elements.

Name ________________________________ID________________________ Section______ CR58_______

8. ____ If a vector space has at least two different subspaces, then it has many infinitely many subspaces.

9. ____ If a coefficient matrix has its column rank equal to its row rank, the corresponding linear system always has a unique solution.

10. ____ For any non-empty sets A and B that $[A] \cap [B] \neq \emptyset$, it is possible to find a vector $\vec{v}$ that is a member of a basis of $[A]$ and of a basis of $[B]$.

11. ____ Let column vectors a, b and c form a basis of $\Re^n$, $n \geq 3$. If the three columns of the $n \times 3$ matrix A are a, b and c, then the system $Ax = 0$ has only the trivial solution.

12. ____If 3x3 matrices A and B have the row rank of 2, then AB also has the row rank of 2.

13. ____Row operations do not change the column space.

14. ____Let A and B be two subspaces of C. If the dimensions of A and B are 3, then the dimension of the subspace $A \cap B$ is 3 as well.

15. ____The linear system $Ax = b$ has a solution if and only if b is in the column space of A

16. ____ If A is a matrix such that the linear system $Ax = 0$ has the unique solution $= 0$, then A must be a square matrix.

17. ____ Every line in $\Re^2$ is a subspace of $\Re^2$.

18. ____ The vectors $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$ form a basis for $\Re^3$.

19. ____ The vectors $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ 1 \\ 1 \\ \pi \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ span $\Re^4$.

20. ____ If A is an n x m matrix, then the set of vectors x in $\Re^m$ such that $Ax = 0$ is a subspace of $\Re^m$.

Name ________________________________ID________________________ Section______ CR58_______

Part II

Fill in your answer in the provided space for each question. (2 point each.)

1. Let $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right)$ be an augmented matrix, its solution set is ___________________

2. If $\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ a \end{pmatrix} \rangle$ is a basis of $\mathfrak{R}^2$, the value of a cannot be________________________________

3. We need at least _____ steps of Gauss's Method operation to make the matrix $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$ be in a reduced echelon form.

4. Let $A = \left\{ \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 32 \\ 8 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 9 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix} \right\}$. Find a set B such that B has 3 elements and $B \subset A$ and $[B] \neq [A]$.

   B is __________________________________

5. Let $S = \{(x, y) | x, y \in \mathfrak{R}\}$ and let the operation "+" and "." be defined as follow; $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$, $r(x, y) = (r(x + y), 1)$. Show that S, with the defined operation, is NOT a vector space.

6. The shortest distance between the origin and the plane $z = 2x + 2y + 3$ is ___________.

7. There are ____ different reduced echelon forms for 3x3 matrices.

8. Suppose that a, b and c span a plane through the origin. Let a = (____, 4, 6), b = (4,____,6). We then have c = (14,16,____).

9. If S and T are subspace of a vector space V, find a condition for S and T such that $[S \cap T] \neq [S] \cap [T]$.

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Name ________________________________ID________________________ Section______ CR58_______

10. Give an equation to make the following system inconsistent. $x + 2y + 3z = 4$, $5x + 6y + 7z = 8$

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11. Somsak wants to invest his 2 million baht into Stock Market, Corporate Bond, and Fixed Account with the annual return of 20%, 10%, and 5% respectively. He expects to receive 250,000 baht at the end of the first year. In addition, the amount of investment in Corporate Bond is 200,000 baht more than the amount in Fixed Account. Write a system of linear equations for finding the amount of investment Somsak puts in each investment type.

12. Find a quadratic polynomial $\mathcal{P}_2$ equation which passes through these points (0,0), (1,1), (2,3).

    y =__________________________________

13. The solution of a homogeneous system $Ax = 0$ where $A = \begin{pmatrix} 1 & -2 & 1 & -1 \\ 2 & -3 & 4 & -3 \\ 3 & -5 & 5 & -4 \end{pmatrix}$ is

    ____

14. The vectors $\left\{ \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ \_\_\_ \end{pmatrix} \right\}$ are linearly dependent.

15. The basis for the span of the vectors $\left\{ \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} \right\}$ is ______________________.

16. Consider the set of two column vectors $\left\{ \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}, \begin{bmatrix} y_1 \\ 0 \\ 0 \end{bmatrix} \right\}$, find the third vector that will extends this set to a basis of $\Re^3$. ____________________

17. What is the condition for the solution set of a k-unknowns linear system is all of $\Re^n$?

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Name ________________________________ID_________________________ Section______ CR58_______

18. The span of $\left\{ \begin{bmatrix} -9 \\ 9 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 9 \\ 1 \end{bmatrix} \right\}$ is a hyperplane of $\Re^4$.

19. What results from applying Gauss-Jordan reduction to a nonsingular matrix?

____

20. Rank of the matrix $\begin{bmatrix} 1 & 1 & 5 & 1 & 4 \\ 2 & -1 & 1 & 2 & 2 \\ 3 & 0 & 6 & 0 & -3 \end{bmatrix}$ is ____________________.

Part III

Solve each question in the provided space. (15 points each.)

1. Consider $\mathcal{P}_2$, find the basis of $\{x^2 - x + 3, 2x^2 + x + 5, x^2 + 5x + 1\}$

Name ________________________________ID________________________ Section______ CR58_______

2. For the following linear system of equations, write down its augmented matrix, solve the system using the Gauss-Jordan reduction method to obtain the reduced echelon form, identify the leading and free variables and describe the solution set (identify particular and homogeneous part of the solution).

$$\begin{aligned} w + 3x + 3y + 2z &= 1 \\ 2w + 6x + 9y + 5z &= 5 \\ -1w - 3x + 3y &= 5 \end{aligned}$$

Name ____________________________________ID___________________________ Section______ CR58_______

3. Let $S = \{a, b\}$ and let $r \cdot x = x$ where $x \in S$ and $r \in \Re$. Let the following table describe the operation "+" of a member of S where the value in row x, column y is the result of $x + y$. Show that S is a vector space

	a	b
a	b	a
b	a	b

Name ___________________________________ID___________________________ Section______ CR58_______

4. Let $A = \left\{ \begin{pmatrix} 2 \\ -2 \\ 6 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 8 \end{pmatrix}, \begin{pmatrix} -1 \\ 6 \\ -7 \end{pmatrix} \right\}$ and $B = \left\{ \begin{pmatrix} 6 \\ 9 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 7 \\ 1 \end{pmatrix} \right\}$. Show that $[A] = [B]$