FACULTY OF ENGINEERING
CHULALONGKORN UNIVERSITY
2110201 Computer Engineering Mathematics
YEAR II, Second Semester, Final Examination, March 3, 2014, 13:00 – 16:00

Name____________________________________ID | | | | | | | | |2|1| CR58_________

Instructions

1. There are 43 questions, with the total number of 8 pages
2. Write down your name and ID on top of every page.
3. Write down your answer only in the provided space.
4. No documents or calculator are allowed inside the exam room. You must not bring any part of this exam paper outside. This exam paper is a government's property. Violators will be prosecuted under a criminal court.
5. A student who wishes to leave the exam room before the end of the exam period must raise their hand and ask for permission before leaving the room. Student must leave the room in orderly manner.
6. Once the time is expired, students must stop writing and must remain seated quietly until the proctors collect all the exam papers or given exam booklets. Only then, the student will be allowed to leave the room in orderly manner.
7. Any student with an act of cheating according to the Faculty of Engineering Announcement regarding the exam regulations is subjected to receive an F and at least 1 semester academic suspension.

I acknowledge all instructions above.

Student's Signature…………………………………….

Part I

True or False. (One point each. A wrong answer is subject to one point deduction.)

1. ____ Let $t\colon \mathbb{R}^3 \to \mathbb{R}^3$ and $s\colon \mathbb{R}^3 \to \mathbb{R}^3$ are linear maps such that the dimension of the range of both s and t is 2. The dimension of the range of $t \circ s$ is also 2.
2. ____ The dimension of the null space of any isomorphism is always 1.
3. ____ Let T be a matrix representation of a homomorphism t under the standard basis. When T is reduced to a row echelon form, the number of the free variable equals to the dimension of the null space of t.
4. ____ Let T be a diagonalizable matrix such that $T = PSP^{-1}$ and the eigenvalues of T are all nonzero real values. The matrix TS is always invertible.

Name ________________________________ID________________________ Section______ CR58_______

5. ____ Let t and s be linear maps and each of them is not an isomorphism. Assume that $(t \circ s)(x) = x$, the matrix representation of $t \circ s$, under any pair of bases, is always invertible.

6. ____ Any four vectors in $\mathbb{R}^3$ must span $\mathbb{R}^3$.

7. ____ There exists a 4x3 matrix A such that $\mathrm{K}(A) = \mathcal{R}(A)$ where $\mathrm{K}(A)$ is kernel of A and $\mathcal{R}(A)$ is range space of A.

8. ____ Let A is an n x n matrix with eigenvalue λ and corresponding eigenvector $\boldsymbol{v}$. Then $3\boldsymbol{v}$ is also an eigenvector of A with an eigenvalue 3λ .

9. ____ If A and B are similar matrixes, then $\det(A) = \det(B)$.

10. ____ If A is diagonalizable, then there is a unique diagonal matrix that is similar to A.

11. ____ Every $n \times n$ matrix with rank lower than *n* has at least one eigenvalue equal to zero.
12. ____ Elementary row operations preserve matrix equivalence and matrix similarity.

13. ____ For a nonsingular $n \times n$ matrix *A*, if *x* is an eigenvector of *A* then *x* is also an eigenvector of A^{-1}.

14. ____ Every change of basis matrix has at least one real eigenvalue.
15. ____ A homomorphism from $\mathbb{R}^3$ to $\mathbb{R}^3$ that can be represented by a matrix with three different real eigenvalues must have nullity equal to zero.
16. ____ Let *t* be an isomorphism from P_0 to *R*, it is obtained that for some non-zero real *k*, *t* has the form *t*(a) = *ka*.
17. ____ There exists a singular matrix that cannot have an isomorphism between row and column spaces.
18. ____ Any two matrices *M* and *N* with the same dimension and the same rank *k*, they are matrices equivalent.
19. ____ For two $n \times n$ matrices *A* and *B*, if Rank(*AB*) = Rank(*A*) = Rank(*B*) > 0, then either *A* or *B* is nonsingular.
20. ____ Give a non-identity matrix with the property that $A^T = A^{-1}$. We can conclude that the determinant of *A* is either 1 or -1.

Part II

Fill in your answer in the provided space for each question. (2 point each.)

Name ________________________________ID________________________ Section______ CR58_______

1. Write the basis of the null space of the transformation $T = \begin{bmatrix} 42 & -42 \\ 42 & -42 \end{bmatrix}$

2. A matrix H of size 3×3 is equivalent only to itself but not any other matrix. Write the matrix H

3. Let $h: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map such that $h\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = \begin{pmatrix} c \\ a \\ b \end{pmatrix}$ and $Rep_{\mathcal{E},\mathcal{E}}(h) = H$, find a change of basis matrix P such that $H = P^{-1}IP$

4. Let $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & 0 \\ 0 & 8 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 6 & -4 & x \end{bmatrix}$. The value of x must be ___ to make A singular.

5. Let $v_1 = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, v_2 = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$ and $v_3 = \begin{pmatrix} g \\ h \\ i \end{pmatrix}$ be a column vector in $\mathbb{R}^3$ and $v_1 \cdot v_2 = 0, v_2 \cdot v_3 = 0, v_1 \cdot v_3 = 0$. Find the invert of $A = [v_2 \quad -v_3 \quad v_1]$

6. Let T_{6x5} be a matrix representation of a linear map such that the kernel of T has dimension 3, thus the map's rank is ______

7. The line that best fits the data points (0,0), (1,1), (2,3) in the least-squares sense is

   y = _____ + _____x

8. Let $\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = 7$. Then, $\det\left(\begin{bmatrix} 2a & 2b & 2c \\ 3d-a & 3e-b & 3f-c \\ 2g & 2h & 2i \end{bmatrix}\right) =$______.

9. Let A is a diagonalizable matrix with characteristic polynomial $\lambda^2(\lambda-1)(\lambda+1)(\lambda+2)^3$. The size of the matrix A is ___x____ .

10. Eigenvectors of $A = \begin{bmatrix} 2 & 4 & 3 \\ -4 & -6 & -3 \\ 3 & 3 & 1 \end{bmatrix}$ are

11. If $A = \begin{bmatrix} 3 & 5 & 8 \\ 0 & 1 & 4 \\ 0 & 0 & 10 \end{bmatrix}$ then A^5 is similar to $\begin{bmatrix} 1 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & y \end{bmatrix}$ where x = ______ and y = ______

12. If A is a diagonalizable $n \times n$ matrix such that $A^2 = I_n$ where I_n is the $n \times n$ identity matrix. Find the eigenvalue(s) of A ____________

13. Suppose that matrix A is similar to matrix B, that is $B = SAS^{-1}$ for a change of basis matrix S. Suppose further that x is an eigenvector of A associated with eigenvalue λ. Is λ also an eigenvalue of B? ____ (yes or no). If it is, is x the associated eigenvector?____ (yes or no). If not, write the formula for the associated eigenvector in terms of S and x . ___________________

14. (4 points!) Find one eigenvalue of this matrix and its associated eigenvector. $\begin{bmatrix} 10 & 34 & 12 & 32 \\ 41 & 17 & 2 & 28 \\ 23 & 35 & 11 & 19 \\ 7 & 11 & 31 & 39 \end{bmatrix}$

15. The inverse of the matrix $\begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ is _________________________________.

16. Find the formula for the n^{th} power of the matrix $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$. ___________________.

17. Show that in the space $\mathbb{R}^3$ the vector x = (1, 1, 0), y = (0, 1, 2), and z = (3, 1, -4) are linearly dependent by finding scalars α and β such that $\alpha x + \beta y + z = 0$.
α = ______________ and β = _____________.

18. Let u = (2, 0, -1), v = (3, 1, 0), and w = (1, -1, c) where $c \in \mathbb{R}$. The set { u, v, w } is a basis for $\mathbb{R}^3$ provided that c is not equal to ________________.

19. The characteristic polynomial of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ is $\lambda^p(3-\lambda)^q$ where p = _______ and q = ________.

Part III

Solve each question in the provided space. (15 points each.)

1. For the following matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$
   a. Write its reduced echelon form.
   b. Calculate A^{-1}
   c. Calculate $\det(A)$

Name ________________________________ID________________________ Section______ CR58_______

2. Let $A = \begin{pmatrix} 0 & 0 & 0 & 9 \\ 0 & 0 & 9 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

   a. Find all eigenvalues with their associated eigenvector of A

   b. Find matrix P and a diagonal matrix D such that $P^{-1}AP = D$

Name ________________________________ID_________________________ Section______ CR58_______

3. Let $B =< 2{,}3 + x^2, 1 - x >$, $\hat{B} =< 4{,}2x, 2 + 4x - 2x^2 >$ and let
$D = \langle [1 \quad 0 \quad 0 \quad 0], [0 \quad 1 \quad 0 \quad 0], [0 \quad 0 \quad 0 \quad 1], [0 \quad 0 \quad 1 \quad 0] \rangle$
$\hat{D} = \langle [1 \quad 1 \quad 1 \quad 1], [1 \quad 1 \quad 0 \quad 0], [1 \quad 0 \quad 1 \quad 1], [0 \quad 1 \quad 1 \quad 0] \rangle$
   a. Show that B is a basis of $\mathcal{P}_2$
   b. Find $\mathrm{Rep}_{D,\hat{D}}(\mathrm{id})$
   c. Let $h: \mathcal{P}_2 \to \mathcal{M}_{1\times 4}$ be a homomorphism such that $\mathrm{Rep}_{B,D}(h) = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 3 & -2 \end{bmatrix}$. Find $\mathrm{Rep}_{\hat{B},\hat{D}}(h)$.

4. Let t be a homomorphism from $\mathbb{R}^2$ to $\mathbb{R}^2$ with nullity equal to 1 and $t\left(\begin{pmatrix} 2 \\ 8 \end{pmatrix}\right) = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$. Let $T = \text{Rep}_{B,B}(t)$ such that one eigenvalue of the matrix T is 5 and its associated eigenvector is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(a) Find $t\left(\begin{pmatrix} -3 \\ 7 \end{pmatrix}\right)$. (b) Find T. Can it be uniquely determined?