FACULTY OF ENGINEERING
CHULALONGKORN UNIVERSITY
2110201 Computer Engineering Mathematics
YEAR II, Second Semester, Final Examination, February 22, 2013, 13:00 – 16:00

Name___________________________________ID | | | | | | | | |2|1| CR58_________

**Instructions**

1. There are 50 questions, with the total number of 8 pages
2. Write down your name and ID on top of every page.
3. Write down your answer only in the provided space.
4. No documents or calculator are allowed inside the exam room. You must not bring any part of this exam paper outside. This exam paper is a government's property. Violators will be prosecuted under a criminal court.
5. A student who wishes to leave the exam room before the end of the exam period must raise their hand and ask for permission before leaving the room. Student must leave the room in orderly manner.
6. Once the time is expired, students must stop writing and must remain seated quietly until the proctors collect all the exam papers or given exam booklets. Only then, the student will be allowed to leave the room in orderly manner.
7. Any student with an act of cheating according to the Faculty of Engineering Announcement regarding the exam regulations is subjected to receive and F and at least 1 semester academic suspension.

I acknowledge all instructions above.

Student's Signature…………………………………….

# Part I

**True or False**. One point each. A wrong answer is subject to one point deduction.

1. ____ If we do Gauss's Method in different ways, we will get the same number of free variables but those free variables might be different.
2. ____ Homogenous system always has solution(s).
3. ____ The following system has a unique solution
$$a + b = 5$$
$$3a + 2c = 3$$
$$5a + 2b + 2c = 6$$
4. ____ We can view the general solution set of any linear system as a linear surface passing through the origin.
5. ____ The following two matrices are row equivalent.
$$\begin{bmatrix} 10 & 5 & 10 \\ 0 & 5 & 6 \\ 2 & 4 & 2 \end{bmatrix} \quad \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$
6. ____ The set $C = \{ a + b\,\pi \mid a, b \in \mathbb{R} \}$ is a subspace of $\mathbb{R}$?
7. ____ Every vector space must have a subspace which is neither trivial space (singleton set of a zero vector) nor itself.

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8. ____ For any different subspaces $A$ and $B$ of the vector space $C$, it is true that $A \cup B$ is also a subspace of $C$.
9. ____ For any different subsets $A$ and $B$ of the vector space $C$, $[A] \cap [B]$ is a subspace of $C$.
10. ____ No linearly independent set contains the zero vector.
11. ____ In any vector space, a sequence of vectors is a basis of the vector space if and only if each vector in the space can be expressed as a linear combination of vectors in that sequence.
12. ____ It is true that, for a linear function $f$, $cf(v_1 + v_2 + v_3) = f(cv_1) + f(cv_2) + f(cv_3)$ when $c$ is a scalar and $v_i$ is a vector.
13. ____ It is possible that a function $f: P_4 \to P_3$ can be uniquely described by providing only four pairs of a mapping between $v_i \in P_4$ and its image $f(v_i) \in P_3$, such that all $v_i$ are distinct.
14. ____ A row operation that swaps two rows of a linear system having $n$ unknown can be always represented by a permutation matrix with $n$ columns.
15. ____ Every isomorphic function has an inverse.
16. ____ Consider the identity function that maps any vector to itself. When this function is written in a matrix form, the matrix always consists of only 1 and 0.
17. ____ Any change of basis matrix has its rank equal to the number of its columns.
18. ____ When a matrix is invertible, it is possible that it represents a function whose nullity is not zero.
19. ____ A matrix having a number of columns more than a number of rows cannot represent a linear transform.
20. ____ Function : $P_2 \to P_3$ , $f(a + bx + cx^2) = 1 + ax + bx^2 + (a + b)x^3$ is a linear map.
21. ____ Two different diagonal matrices cannot belong to the same similarity class.
22. ____ The reduced echelon of a nonsingular matrix always has 1 as an eigenvalue.
23. ____ There is no matrix equivalent class that has only one member.
24. ____ Let $T$ be a square matrix and $I$ be the identity matrix of the same size. If columns of $T + kI$ are linearly dependent for some scalar $k$, then $k$ is an eigenvector of $T$.
25. ____ The determinant function is linear.

## Part II

**Fill in your answer in the provided space for each question**. One point each.

1. What is the circle that passes through (3, -1) (-2, 4) (6,8)
   Given the standard equation of a circle is $x^2 + y^2 + ax + by + c = 0$

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2. What is the general solution set of this system

$$a + b + 3c + 2d = 1$$
$$a + c + d = -1$$
$$2a - b + d = 3$$

3. What value(s) of "$r$" that makes this system has no solution?

$$2a + 5b + c = 10$$
$$3a + 2b + 4c = 7$$
$$ra + 7b + 5c = 17$$

4. What is "$k$" that makes these two vectors orthogonal?

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ k \end{bmatrix}$$

5. Somsak wants to invest his 2 million baht into Stock Market, Corporate Bond, and Fixed Account with the annual return of 15%, 6%, and 3% respectively. He expects to receive 240,000 baht at the end of the first year. In addition, the amount of investment in Bond is 200,000 baht more than the amount in Fixed Account.

   Write a system of linear equations for finding the amount of investment Somsak puts in each investment type.

6. If $S$ and $T$ are subspace of a vector space $V$, find a condition for $S$ and $T$ such that $[S \cap T] \neq [S] \cup [T]$.

7. Find a structure (set, addition and scalar multiplication operations) that is closed under linear combinations, and yet is not a vector space.

8. Find a condition, in term of exactly one equation, that the subset of $\mathbb{R}^3 = \{ \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} d \\ e \\ f \end{pmatrix}, \begin{pmatrix} g \\ h \\ i \end{pmatrix} \}$ becomes linearly **dependent** set.

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9. Find a basis for the subspace $\{ a_0x^3 + a_1x^2 + a_2x + a_3 \mid a_2 - 2a_1 = a_0 \text{ and } a_3 + 2a_1 = a_2\}$ of the space of polynomial $P_3$ .

10. Find an $n \times n$ non-symmetric matrix ($A \neq A^T$) such that the row space is equal to the column space, if there exists such matrix.

11. The following matrix $H$ represents, with respect to the standard basis, a function $h$ that maps from $\mathbb{R}^5$ to $\mathbb{R}^3$. Answer the following questions

$$H = \begin{pmatrix} 1 & 2 & 3 & 0 & 1 \\ 4 & 5 & 6 & 1 & 0 \\ 5 & 7 & 9 & 1 & 1 \end{pmatrix}$$

  a. What is the nullity of $h$? ____
  b. The dimension of the row space of $H$ is ____
  c. What is the column rank of $H$? ____

12. If a function map from $\mathbb{R}^9$ to $\mathbb{R}^5$ and this function is on-to, what is the nullity of this function? ____

13. Let $B$ be a matrix with 3 rows and 2 columns. Let $C$ be a matrix with 2 rows and 3 columns. If a matrix $A = CB$ is invertible.
  a. What is the nullity of the function that is represented by the matrix $B$? ____
  b. What is the nullity of the function that is represented by the matrix $C$? ____

14. What value of $k$ in the following matrix to make it a change of basis matrix for the vector space $\mathbb{R}^3$.

$$\begin{pmatrix} 3 & 3 & 0 \\ 2 & 4 & -1 \\ k & 6 & 8 \end{pmatrix}$$

15. We would like to perform a series of three row operations on a linear system with 5 equations. The operations are $\xrightarrow{\rho_1 \leftrightarrow \rho_5}$ followed by $\xrightarrow{\rho_3 \leftrightarrow \rho_4}$ and $\xrightarrow{4\rho_3 + \rho_1}$. Write one elementary reduction matrix that represents this series of operations.

16. Let $f: V \to V$ be a linear transformation and let $v_1$ and $v_2$ be members of $V$. If $f(v_1 + 2v_2) = 3v_1 - v_2$ and $f(v_1 - v_2) = 2v_1 - 4v_2$, what is the value of $f(v_1)$ and $f(v_2)$, in terms of $v_1$ and $v_2$.

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17. Assume that $M$ is a $5 \times 3$ matrix. If we take each column of this matrix as a column vector, assume that the set of these vectors are linearly independent. What is the dimension of the row space? _____
18. Consider a linear system with 4 unknowns. Suppose that the dimension of the row space of the linear equations that describe this system is 1. What is the number of free variables in the solution set? ____
19. How many 5x5 matrix equivalent classes are there ? _____
20. det $\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$ = $(b-a)$ ( ___ - ___)(___ - ___)

## Part III

**Solve each question in the provided space**. Six points each.

1. For the following matrix $A = \begin{pmatrix} 1 & 0 & 1 & 2 \\ 2 & 1 & 2 & 2 \\ 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 \end{pmatrix}$
   a. Write its reduced echelon form.
   b. Calculate $A^{-1}$.
   c. Calculate $\det(A)$

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2. Suppose the solution of a linear system $A\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = b$ is $\begin{pmatrix} 3 \\ 4 \\ 1 \\ 3 \end{pmatrix} + k\begin{pmatrix} 3 \\ 2 \\ 1 \\ 0 \end{pmatrix} + m\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$, $k, m \in \mathbb{R}$

   a. Write the augmented matrix $[A \mid b]$ of this system in the reduced echelon form.
   b. List 3 different particular solutions of this system.
   c. What is the dimension of the null space of this system?

3. Let $B = <1, x-1, (x-1)^2>$, $D = <x+1, x-1, \mathrm{x}^2>$ and $E = <x^2, x+1, x>$

   a. Show that $B$ forms a basis for the vector space $P_2$.
   b. Find $\mathrm{Rep}_{B,D}(id)$.
   c. Let $\mathrm{Rep}_{B,E}(t)$. = $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix}$. Find $\mathrm{Rep}_{E,D}(t)$

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4. Let $A = \begin{pmatrix} 9 & 0 & 0 \\ 3 & 4 & 0 \\ 5 & 7 & 2 \end{pmatrix}$,

   a. Find all eigenvalues with their associated eigenvector of $A$

   b. Find matrix $P$ and a diagonal matrix $D$ such that $P^{-1}AP = D$

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5. Find two of the eigenvectors of this matrix.

$$\begin{pmatrix} 1 & 2 & 1 & 3 & 0 & 1 \\ 2 & 4 & 2 & 6 & 1 & 4 \\ 1 & 2 & 3 & 5 & 0 & 3 \\ 2 & 4 & 4 & 8 & 1 & 6 \\ 1 & 2 & 5 & 7 & 0 & 5 \\ 2 & 4 & 6 & 10 & 1 & 8 \end{pmatrix}$$